Fine tuning and the ratio of tensor to scalar density fluctuations from cosmological inflation

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Abstract

The form of the inflationary potential is severely restricted if one requires that it be natural in the technical sense, i.e. terms of unrelated origin are not required to be correlated. We determine the constraints on observables that are implied in such natural inflationary models, in particular on r, the ratio of tensor to scalar perturbations. We find that the naturalness constraint does not require r to be lare enough to be detectable by the forthcoming searches for B-mode polarisation in CMB maps. We show also that the value of r is a sensitive discriminator between inflationary models.

The nature of the density perturbations originating in the early universe has been of great interest both observationally and theoretically. The hypothesis that they were generated during an early period of inflationary expansion has been shown to be consistent with all present observations. The most discussed mechanism for inflation is the 'slow roll' of a weakly coupled 'inflaton' field down its potential — the near-constant vacuum energy of the system during the slow-roll evolution drives a period of exponentially fast expansion and the density perturbations have their origin as quantum fluctuations in the inflaton energy density.

In such models the detailed structure of the density perturbations which give rise to the large scale structure of the universe observed today depends on the nature of the inflationary potential in the field region where they were generated. Boyle, Steinhardt and Turok [1] have argued that "naturalness" imposes such strong restrictions on the inflationary potential that one may derive interesting constraints on observables today. They concluded that in theories which are "natural" according to their criterion, the spectral index of the scalar density perturbations is bounded as $n_\mathrm{s} < 0.98$, and that the ratio of tensor-to-scalar perturbations satisfies $r > 0.01$ provided $n_\mathrm{s} > 0.95$, in accord with then current measurements [2]. Such a lower limit on the amplitude of gravitational waves is of enormous interest as there is then a realistic possibility of detecting them as 'B-mode' polarisation in CMB sky maps (see e.g. [3]) and thus verifying a key prediction of inflation.

Of course these conclusions are crucially dependent on the definition of naturalness. In this paper we re-examine this important issue and argue that the criterion proposed by Boyle *et al* does not capture the essential aspects of a *physically* natural theory. We propose an alternative criterion that correctly reflects the constraints coming from underlying symmetries of the theory and we use this to determine a new bound on r that turns out to quite opposite to the previously inferred one. We emphasise that our result, although superficially similar to the 'Lyth bound' [4], follows in fact from different considerations and in particular makes no reference to how long inflation lasts.

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Inflation predicts a near scale-invariant spectrum for the scalar and tensor fluctuations, the former being in reasonable agreement with current observations. Here we explore the predictions for *natural* models involving a single inflaton at the time the density perturbations are produced. Models with two or more scalar fields affecting the density perturbations require some measure of fine tuning to relate their contribution to the energy density, whereas the single field models avoid this unnatural aspect. In order to characterize the inflationary possibilities in a model independent way it is convenient to expand the inflationary potential about the value of the field ϕ_H just at the start of the observable inflation era, ~ 60 e-folds before the end of inflation when the scalar density perturbation on the scale of our present Hubble radius [1] was generated, and expand in the field $\phi^* \equiv \phi - \phi_\mathrm{H}$ [5]. Since the potential must be very flat to drive inflation, ϕ^* will necessarily be *small* while the observable density perturbations are produced, so the Taylor expansion of the potential will be dominated by low powers of ϕ^*:

$$V(\phi^*) = V(0) + V'(0)\phi^* + \frac{1}{2}V''(0)\phi^{*2} + \dots \tag{1}$$

The first term $V(0)$ provides the near-constant vacuum energy driving inflation while the ϕ^*-dependent terms are ultimately responsible for ending inflation, driving ϕ^* large until higher-order terms violate the slow-roll conditions. These terms also determine the nature of the density perturbations produced, in particular the departure from a scale-invariant spectrum.

The observable features of the primordial density fluctuations can readily be expressed in terms of the coefficients of the Taylor series [5]. It is customary to use these coefficients first to define the slow-roll parameters ϵ and η [6] which must be small during inflation:

$$\epsilon \equiv \frac{M^2}{2}\left(\frac{V'(0)}{V(0)}\right)^2 \ll 1, \qquad |\eta| \equiv M^2\left|\frac{V''(0)}{V(0)}\right| \ll 1, \tag{2}$$

where M is the reduced Planck scale, $M = 2.44 \times 10^{18}$ GeV. In terms of these the spectral index is given by

$$n_\mathrm{s} = 1 + 2\eta - 6\epsilon, \tag{3}$$

the tensor-to-scalar ratio is

$$r = 16\epsilon, \tag{4}$$

and the density perturbation at wave number k is

$$\delta_\mathrm{H}^2(k) = \frac{1}{150\pi^2}\frac{V(0)}{\epsilon M^4}. \tag{5}$$

Finally the 'running' of the spectral index is given by

$$n_\mathrm{r} \equiv \frac{\mathrm{d}n_\mathrm{s}}{\mathrm{d}\ln k} = 16\epsilon\eta - 24\epsilon^2 - 2\xi, \tag{6}$$

where

$$\xi \equiv M^4\frac{V'V'''}{V^2}. \tag{7}$$

At this stage we have four observables, n_s, n_r, δ_H and r and four unknown parameters $V(0)$, $V'(0)$, $V''(0)$ and $V'''(0)$ which, for an arbitrary inflation potential, are independent. However for natural potentials these parameters are related, leading to corresponding relations between the observables. Observational confirmation of such relations would provide evidence for the underlying potential, hence crucial clues to the physics behind inflation.

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[1] Numerically this is $H_0^{-1} \simeq 3000h^{-1}$ Mpc, where $h \equiv H_0/100$ km s^{-1} Mpc^{-1} ~ 0.7 is the Hubble parameter. The density perturbation is measured down to ~ 1 Mpc, a spatial range corresponding to ~ 8 e-folds of inflation.

As discussed above we are considering the class of natural models in which a single inflaton field dominates when the density perturbations relevant to the large-scale structure of the universe today are being produced.[2] In classifying "natural" inflation, Boyle *et al* imposed a set of five conditions [1]:

1. The energy density (scalar) perturbations generated by inflation must have amplitude $\sim 10^{-5}$ on the scales that left the horizon ≈ 60 e-folds before the end of inflation;

2. The universe undergoes at least $N > 60$ e-folds of inflation;

3. After inflation, the field must evolve smoothly to an analytic minimum with $V = 0$;

4. If the minimum is metastable, then it must be long-lived and V must be bounded from below;

5. Inflation must halt and the universe must reheat without spoiling its large-scale homogeneity and isotropy.

They proposed that the level of fine-tuning for potentials satisfying the above conditions should be measured by the integers $Z_{\epsilon,\eta}$ that measure the number of zeros that ϵ and η and their derivatives undergo within the last 60 e-folds of inflation [1]. Here we argue that such a measure does *not* capture the essential character of physical naturalness.

At a purely calculational level this is illustrated by the fact that it is necessary to impose an (arbitrary) cut-off on the number of derivatives included in the criterion.[3] This is necessary because ϵ and η are defined in terms of the ratio of first or second order derivatives of the potential to the potential itself, so *all* higher order derivatives must be considered separately when counting the total number of zeros. The difficulty follows from the observation that, as far as naturalness is concerned, it is the inflaton potential that is the primary object, being restricted by the underlying symmetries of the (effective) field theory describing the inflaton dynamics. As stressed by 't Hooft [7], a *natural* theory is one in which all terms in the Lagrangian allowed by the underlying symmetries of the theory are present, with no relations assumed between terms unrelated by the symmetries.

It is important however to note that such natural potentials do *not* preclude significant contributions from unrelated terms. Indeed such contributions are inevitable if, for example, the inflation field is moving from small to large field values. For small field values the lowest allowed power in the inflaton field is likely to be the most important, but at larger field values higher powers will ultimately dominate. For this reason the last four conditions have a different character to the first in that they involve the end of inflation when naturalness does *not* require that a single term in the inflation potential should dominate. For example in inflationary models with the inflaton rolling from small to large field values, the higher powers can cause the potential to evolve smoothly to an analytic minimum with $V = 0$ or govern the properties of an unstable minimum. Similarly it may be these higher powers that cause inflation to halt and the universe to reheat without affecting the predictions for the observable density perturbation. Given the freedom there is in choosing these higher powers (non-renormalisable terms in the effective field theory description), it is *always* possible to find a model in which the end of inflation is satisfactory without violating the naturalness constraints [5]. On the other hand the range of ϕ^* relevant during the production of density perturbations is quite small (corresponding to only 8 of the ~ 60 e-folds of inflation) and so it is reasonable to suppose that unrelated terms do not simultaneously contribute significantly to the generation of the observed density perturbation. Although we have made this argument in the context of 'new inflation' models where the inflaton field evolves from low to high values, similar

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[2]This does not exclude 'hybrid inflation' models in which additional fields play a role at the end of inflation.

[3]Boyle *et al* imposed the cut-off at 15 derivatives (L. Boyle, private communication).

considerations apply to the other natural models. For the case where the underlying symmetry is a Goldstone symmetry it is still possible to change the end of inflation in a natural way through the effect of a second 'hybrid' field. Given these considerations we do not need to impose the last four conditions when determining the phenomenological implications of natural inflationary models. However we will comment on how these conditions can indeed be satisfied for the various classes of inflation potential.

Our definition of naturalness is the standard one in particle physics [7], viz. pertaining to a potential whose form is guaranteed by a symmetry. This should apply at the time the observable density perturbations are being produced. What form can such natural potentials take? The relevant symmetries that have been identified capable of restricting the scalar inflaton potential are relatively limited. The most direct are Abelian or non-Abelian symmetries, either global or gauge, and continuous or discrete. For a single field inflation model these will either limit the powers of the inflaton field that may appear in the potential or, if the inflaton is a pseudo-Goldstone mode, require a specific form for the potential. Less direct constraints occur in supersymmetric theories where the scalar inflaton field is related to a fermionic partner. In this case chiral symmetries of the associated fermion partner and R-symmetries may further restrict the form of the potential.[4] As observed earlier [8], such symmetries are very promising for eliminating the fine-tuning problem in inflationary potentials because they can forbid the large quadratic terms in the inflaton field that, even if absent at tree level, arise in radiative order in non-supersymmetric theories (unless protected by a Goldstone symmetry).

We turn now to a discussion of the observable implications of the natural inflation models. In this we find it useful to classify the models into two classes, namely those involving small, sub-Planckian field values only and those that require large, super-Planckian, field values. Here we use the reduced Planck scale, M, to define the sub- and super- regimes as this is the scale that orders typical higher order terms in supergravity. In the small-field models we allow for the possibility of higher order terms which can dominate as the vacuum expectation value of the inflaton field becomes large. In the large field models it is necessary to *forbid* such higher order terms since they would otherwise dominate the potential and there should be an underlying symmetry to enable this to be done.

1   Small field models

These potentials are of the 'new inflation' form

$$V(\phi) = \Delta^4 \left[1 - \lambda \left(\frac{\phi}{\Lambda} \right)^p \right],\tag{8}$$

with a single power of the inflaton field, ϕ, responsible for the variation of the potential, plus a constant term driving inflation. Such a form does not require fine-tuning as the two terms need not be related and the dominance of a given single power can be guaranteed by a symmetry [8]. Since the slow-roll parameters get no contribution from the constant term their main contribution will necessarily come from the leading term involving the inflaton field and the naturalness condition is trivially satisfied because this is dominated by a single power of ϕ. From Eq.(5) it is clear that δ_{H} is the only observable that depends on Δ, so one can fit its observed value but cannot predict

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[4] In supersymmetric theories it is the superpotential that is constrained by the underlying symmetries. The resulting scalar potential has natural relations between different powers of the inflaton field.

it without a theory for Δ. The slow-roll parameters are given by

$$\eta = -\lambda p(p-1)\left(\frac{\phi_{\mathrm{H}}}{\Lambda}\right)^{p-2}\left(\frac{M}{\Lambda}\right)^2, \tag{9}$$

$$\epsilon = \frac{\lambda^2 p^2}{2}\left(\frac{\phi_{\mathrm{H}}}{\Lambda}\right)^{2p-2}\left(\frac{M}{\Lambda}\right)^2 = \eta^2\frac{1}{2(p-1)^2}\left(\frac{\phi_{\mathrm{H}}}{M}\right)^2, \tag{10}$$

$$\xi = \lambda^2 p^2(p-1)(p-2)\left(\frac{\phi_{\mathrm{H}}}{\Lambda}\right)^{2p-4}\left(\frac{M}{\Lambda}\right)^4 = \eta^2\frac{(p-2)}{(p-1)}. \tag{11}$$

Turning to the other observables let us consider first the cases $p \geq 2$. Note that the naturalness arguments apply only if $\phi/\Lambda < 1$ and hence $|\eta| > \epsilon$. In this case n_{s} is effectively determined by η alone, so the measurement of n_{s} does not impose a lower bound on ϵ. Thus the expectation is that r will naturally be small for this class of models [8]. To quantify this we note that $\eta \simeq (1 - n_{\mathrm{s}})/2$ hence

$$r = 16\epsilon = \eta^2\frac{8}{(p-1)^2}\left(\frac{\phi_{\mathrm{H}}}{M}\right)^2. \tag{12}$$

This implies that any value $0.9 \lesssim n_{\mathrm{s}} \lesssim 1$ can be obtained. Imposing the bound $n_{\mathrm{s}} > 0.95$ following Boyle *et al* [1][5] then requires $r < 0.005$. We emphasise that this makes no explicit reference to the excursion of the field during inflation, as in the 'Lyth bound' [4]. Note that the precise value for r depends here on the value of ϕ_{H} which, as discussed earlier, is determined by the higher order terms that may be present in the potential. Specific examples have been constructed [5] showing that r can be much lower than the bound given above, even as small as 10^{-16}. These results are inconsistent with the *lower* bound quoted by Boyle *et al* [1] and reflect our different physical interpretation of naturalness. Finally the prediction for n_{r} is

$$n_{\mathrm{r}} \simeq -2\xi \simeq -0.001\frac{(p-2)}{(p-1)}. \tag{13}$$

The case $p = 1$ is special since now η and ξ both vanish giving $r = 8(1 - n_{\mathrm{s}})/3 < 0.13$ and $n_{\mathrm{r}} = -2(1 - n_{\mathrm{s}})^2/3 \simeq 10^{-3}$. This is the *only* case of a sub-Planckian model yielding a large tensor amplitude and it has been argued [10] that this case cannot be realised in a complete model due to the requirement that the universe should undergo at least ~ 50 e-folds of inflation. The problem is that for this case $\epsilon = (1 - n_{\mathrm{s}})/6$ is large, limiting the number of e-foldings, which is given by

$$N = \frac{1}{M}\int_{\phi_{\mathrm{H}}}^{\phi_{\mathrm{e}}}\frac{1}{\sqrt{2\epsilon}}\mathrm{d}\phi, \tag{14}$$

where ϕ_{e} is the field value at the end of inflation. For sub-Planckian models $\phi_{\mathrm{e}} \leq M$, hence $N < 1/\sqrt{2\epsilon} = \sqrt{3/(1 - n_{\mathrm{s}})}$. For the case $n_{\mathrm{s}} = 0.95$, which gives the large r value, we have only $N < 8$ e-folds. In this case the effect of higher order terms near the Planck scale does not help as the linear term already contributes too much to the slope of the potential and thus limits the number of e-folds of inflation. The only way out of this is that there should be a subsequent inflationary era which generates $\sim 40 - 50$ additional e-folds of inflation after the ϕ field has settled into its minimum. At first sight this looks like an unnatural requirement. However we have shown elsewhere [11] that in supergravity models it is natural to expect some $\sim 3\ln(M/\Lambda)$ e-folds of 'multiple inflation' due to intermediate scale symmetry breaking along 'flat directions', where Λ^4 is the magnitude of the potential driving this subsequent period of inflation. Taking $\Lambda \sim 10^{11}$ GeV (typical of the supersymmetry breaking scale in supergravity models) one generates ~ 50 e-folds of inflation. Although this two-stage inflationary model appears complicated, it is still natural in the sense discussed above and should not be ignored as a possibility.

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[5]This is slightly more restrictive than the recent WMAP 5-year result: $n_{\mathrm{s}} = 0.963^{+0.014}_{-0.015}$ [9].

2 Large field models

2.1 Chaotic inflation

The simplest potential involves a single power of the inflaton

$$V(\phi) = \lambda \frac{\phi^p}{\Lambda^{p-4}} \ , \tag{15}$$

where we have allowed for the possibility that the scale, Λ, relevant for higher dimensional terms in the effective potential need not be the Planck scale but can correspond to the mass of some heavy states that have been integrated out in forming the effective potential. Expanding around $\phi = \phi_{\mathrm{H}}$ yields

$$\epsilon = \frac{p^2}{2}\left(\frac{M}{\phi_{\mathrm{H}}}\right)^2, \tag{16}$$

$$\eta = p(p-1)\left(\frac{M}{\phi_{\mathrm{H}}}\right)^2, \tag{17}$$

$$\xi = p^2(p-1)(p-2)\left(\frac{M}{\phi_{\mathrm{H}}}\right)^4. \tag{18}$$

The slow-roll conditions, $\epsilon, |\eta| \ll 1$, requires $M/\phi_{\mathrm{H}} \ll 1$ which means that inflation occurs for ϕ above the Planck scale — usually called 'chaotic inflation' [12].[6] In this case, in order to explain why ever higher order terms ϕ^m, $m \to \infty$, do not dominate, it is necessary to have a symmetry which forbids such terms. One such (Goldstone) symmetry has been invoked in a supergravity context [14], although it is not known if this can arise in realistic models. Another recent proposal for a large field potential exploits monodromy in a D-brane setup but contains no Standard Model sector which would give rise to large corrections to the slow-roll parameters [15]. Thus whether such models can actually be realised remains an open question.

What are the observable implications of this potential? As before, δ_{H} is the only observable that depends on λ so one can fit its observed value but lacking a theory for λ this is not a prediction. The other 3 observables are determined in terms of the parameter $x = M/\phi_{\mathrm{H}}$ and the power p.

$$n_{\mathrm{s}} = 1 - x^2 p(p+2), \tag{19}$$

$$r = 8p^2 x^2, \tag{20}$$

$$n_{\mathrm{r}} = -2x^4 p^2(p+2). \tag{21}$$

From this one sees that the ratio of tensor to scalar fluctuations and the running of the spectral index are tightly constrained by the measurement of n_{s}

$$r = \frac{8p}{(p+2)}(1-n_{\mathrm{s}}), \tag{22}$$

$$n_{\mathrm{r}} = -\frac{2}{(p+2)}(1-n_{\mathrm{s}})^2. \tag{23}$$

Note that these results are independent of ϕ_{H} and so, as anticipated above, do not depend on exactly when inflation ends. For the quartic potential $p = 4$, $r = 0.27$ and $n_{\mathrm{r}} \simeq -8 \times 10^{-4}$. The maximum value of r is $8(1-n_{\mathrm{s}}) \simeq 0.4$ with $n_{\mathrm{r}} = 0$.

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[6]In fact "chaotic" actually refers to the initial conditions for inflation and 'chaotic inflation' can also be realised in a small-field model [13].

2.2 Natural inflation

Another class of non-fine-tuned models is based on an approximate Goldstone symmetry [16], often called 'natural inflation' (although it should now be clear that this is not the *only* natural possibility). In this case the potential is not a simple polynomial but has the form

$$V(\phi) = \Delta^4 \left(1 + \cos \frac{\phi}{f} \right). \tag{24}$$

The slow-roll parameters are:

$$\epsilon = \frac{1}{2} \left(\frac{M}{f} \right)^2 \frac{\left(\sin \frac{\phi_{\mathrm{H}}}{f} \right)^2}{\left(1 + \cos \frac{\phi_{\mathrm{H}}}{f} \right)^2}, \tag{25}$$

$$\eta = - \left(\frac{M}{f} \right)^2 \frac{\cos \frac{\phi_{\mathrm{H}}}{f}}{\left(1 + \cos \frac{\phi_{\mathrm{H}}}{f} \right)}, \tag{26}$$

$$\xi = - \left(\frac{M}{f} \right)^4 \frac{\left(\sin \frac{\phi_{\mathrm{H}}}{f} \right)^2}{\left(1 + \cos \frac{\phi_{\mathrm{H}}}{f} \right)^2}. \tag{27}$$

For these to be small we require $f > M$. Unlike the previous case the predictions now depend sensitively on ϕ_{H} and hence on the related value of the field at the *end* of inflation.

If the end of inflation is determined, as has usually been *assumed*, by the steepening of the above potential then ϕ_{H} has a value such that ϵ and η are comparable. In this case ϵ can be close to its slow-roll limit, particularly interesting for tensor fluctuations which can now be large. Imposing the bound $n_{\mathrm{s}} > 0.95$ [9] implies $0.02 < r < 0.2$ [17], the range corresponding to the variation of ϕ_{H} with f for allowed values of f. As with the other models, the running is small, $n_{\mathrm{r}} \sim \mathcal{O}(10^{-3})$.

However it may be more natural for inflation to end much earlier due to a second (hybrid) field. Then ϕ_{H} is reduced so that $\sin(\phi_{\mathrm{H}}/f)$ can be small, hence $|\eta| \gg \epsilon$. In this case r will be (arbitrarily) small, being proportional to ϵ, The running is also very small, $n_{\mathrm{r}} \simeq 12\epsilon\eta$.

3 Conclusions

We have discussed natural possibilities for the inflationary potential. From this it is clear that the gravitational wave signal need not be large enough to be observable as argued by Boyle *et al* [1].

The models considered fall broadly into two classes. The first has ϵ comparable in magnitude to η hence r can saturate the upper bound of 0.4 implied by the slow-roll constraint. A characteristic of these models however is that inflation occurs only at field values higher than the Planck scale and it is not clear if this can be naturally realised. An interesting exception is the 'new inflation' model with a leading *linear* term in the inflaton field which however requires a subsequent period of inflation to create our present Hubble volume.

The second class of models has η larger than ϵ. These are indeed natural but there is no lower bound to r and the upper bound is (unobservably) small. A characteristic of most such models is that inflation occurs at low field values, much below the Planck scale. Examples of this are provided by 'new inflation' where r is bounded from *above* by 0.005 and is usually much below this bound. A large-feld exception to this is a modified form of 'natural inflation' where a hybrid field ends inflation early.

To summarise, in models that are not fine-tuned, the amplitude of density perturbations and the spectral index are not predicted, being determined by free parameters of the model. However

the tensor-to-scalar ratio, r, and the running, $n_{\rm r}$, are determined in terms of the spectral index. The ratio r provides a sensitive discriminator of the natural models but there is no requirement that it be greater than 0.01 even if the spectral index is bounded from below $n_{\rm s} > 0.95$.[7] All the natural models have the running small, $n_{\rm r} \sim (1 - n_{\rm s})^2$, so observation of a much larger value would indicate a departure from naturalness, perhaps because more than one inflaton field is active at the time density perturbations are generated. This in turn would suggest there should be a departure from a near-Gaussian distribution of the perturbations.

4   Acknowledgements

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[7] This conclusion is in agreement with another analysis of specific models [18].